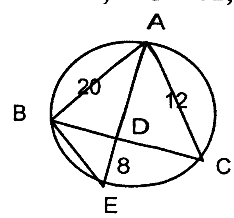
GEOMETRY

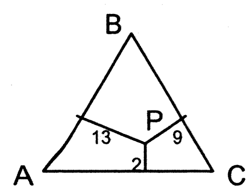
1. The bisector of $\angle A$ of ΔABC intersects the circumcircle of ΔABC at E, and $\overline{AE}$ intersects $\overline{BC}$ at D. If AB = 20, AC = 12, and DE = 8, what is the length of $\overline{AD}$?



2. An equilateral triangle of perimeter 6 sits atop a square of perimeter 8, and the two share a side in common. Line segments of length **x** connect the two vertices of the square that aren't also vertices of the triangle to the vertex of the triangle that isn't also a vertex of the square. What is the value of **x**?
Express your answer in the form $\sqrt{a}+\sqrt{b}$, where a and b are integers.

3. What is the numerical value of ***b*** for which the length of the path from A(0, 2) to B(***b***, 0) to C(c, 10) to D(5, 9) will be a **minimum**? Express your answer as a fraction in simplest form.
[HINT, since the shortest distance between *two* points is a straight line segment, draw $\overline{A'BCD'}$, such that its length is equal to the path $\overline{AB}$ to $\overline{BC}$ to $\overline{CD}$.]

4. **P** is a point in the interior of **equilateral** triangle ABC, such that perpendicular segments from P to each of the sides of ΔABC measure 2 inches, 9 inches and 13 inches. Find the number of square inches in the area of ΔABC, and express your answer in simplest radical form.



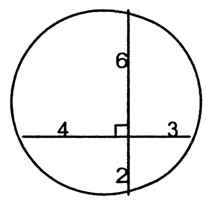
5. Quadrilateral Q has an inscribed circle. If the lengths of 3 consecutive sides of Q are 13, 14, and 15, what is the perimeter of Q? Express your answer in simplest form.

6. Find the area of the convex quadrilateral formed by connecting the points of intersection of the graphs of $xy = 20$ and $x^2 + y^2 = 41$.
Express your answer in simplest form.

7. The lengths of the bases of trapezoid T are 8 and 18. If the lower base angles of trapezoid T are complementary, what is the distance between the midpoints of the upper and lower bases of T? Express your answer in simplest form
Hint: Draw segments parallel to the legs.

8. A circle passes through point A(3, 4) and point B(6, 8) and is tangent to the x axis at point C(k, 0). Find k and express in simplest radical form.
(Hint: extend chord $\overline{AB}$.)

9. Two perpendicular chords intersect in circle O. The lengths of the segments of the longer chord are 6 and 2, while the lengths of the shorter chord are 4 and 3. Find the length of the diameter of circle O, and express it in simplest radical form.

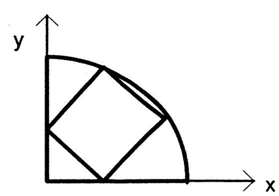


10. Given ΔABC with AB = 14, BC = 15, AC = 13. The length of the shortest altitude is $56/5$. Find the sum of the lengths of the two longer altitudes of ΔABC in simplest form.

11. Given ΔABC, AC = 10, BC = 12. D is on $\overline{AB}$, such that $\overline{CD} \perp \overline{AB}$. Point E lies on $\overline{CD}$, such that AE = 4, EB = x. Compute x.

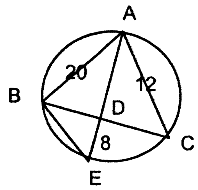
12. The lengths of the diagonals of a rhombus are 16 and 30. Compute the length of an altitude drawn from a vertex to the opposite side.

13. As shown in the diagram below, a square is inscribed in a quadrant of a circle with a radius of 5 inches. Find the number of square inches in the area of the square. Express your answer in simplest form.



Solutions:

1).



AD = x, Triangles ACD and AEB are similar so

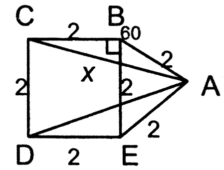
x : 20 = 12 : x + 8

$x^2 + 8x - 240 = 0$

$(x - 12)(x + 20) = 0$

$x = 12$ reject $x = -20$

2.



$$AC^2 = 2^2 + 2^2 - 2 \bullet 2 \bullet 2\cos 150^\circ$$

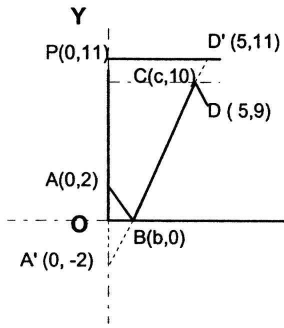
$$= 4 + 4 - 8\left(-\frac{\sqrt{3}}{2}\right) = 8 + 4\sqrt{3}$$

$$AC = \sqrt{8 + 4\sqrt{3}} = \sqrt{8 + 2\sqrt{12}} = \sqrt{6 + 2 + 2\sqrt{6 \bullet 2}}$$

$AC = \sqrt{6} + \sqrt{2}$ **Note:** $\left(\sqrt{6} + \sqrt{2}\right)^2 =$

$$\sqrt{6}^2 + 2\sqrt{6}\sqrt{2} + \sqrt{2}^2 = 6 + 2\sqrt{12} + 2 = 8 + 4\sqrt{3} = AC^2$$

3

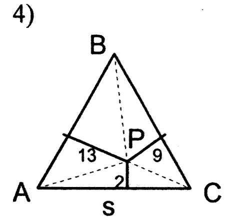


By reflections, the length of $\overline{A'BCD'}$ is equal to the length of the path $\overline{AB}$ to $\overline{BC}$ to $\overline{CD}$.

$\Delta A'OB \sim \Delta A'PD'$

So $\frac{2}{b} = \frac{13}{5}$ thus $b = \frac{10}{13}$

4)



Area triangle APB = (1/2)(13)s

Area triangle BPC = (1/2)(9)s

Area triangle APC = (1/2)(2)s

Area triangle ABC =(24/2)s=12s

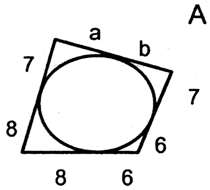
Area of an equilateral Δ ABC = $\frac{s^2}{4}\sqrt{3}$ =12s

So $s = 16\sqrt{3}$

and

Area ΔABC= $\frac{s^2}{4}\sqrt{3} = \frac{(16\sqrt{3})^2}{4}\sqrt{3} = 192\sqrt{3}$

5).



A possible quadrilateral is shown

Sides of quadrilateral Q are tangent segments to the inscribed circle.

So a = 7 and b = 7

Perimeter is 13 + 14 + 15 + 14 = 56

Theorem: For any quadrilateral with an inscribed circle, the sum of the lengths of each pair of opposite sides of the quadrilateral is the same.

6) $x^2 + \left(20/x\right)^2 = 41$ or

$x^2 + 400/x^2 = 41$

So $x^4 - 41x^2 + 400 = 0$ or

$(x^2 - 16)(x^2 - 25) = 0$

Thus x = ±4, x =±5 giving the solutions:

(4, 5), (-4, -5), (5, 4), and (-5,-4)

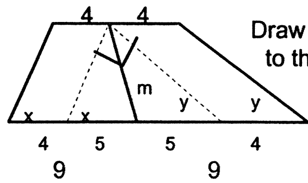
These form a rectangle, whose dimensions are:

$d_{(5,4)(4,5)} = \sqrt{(5-4)^2 + (4-5)^2} = \sqrt{2}$

$d_{(5,4)(-4,-5)} = \sqrt{(5-(-4))^2 + (4-(-5))^2} = \sqrt{162}$

The area is therefore $(\sqrt{2})(\sqrt{162}) = \sqrt{324}$ or 18

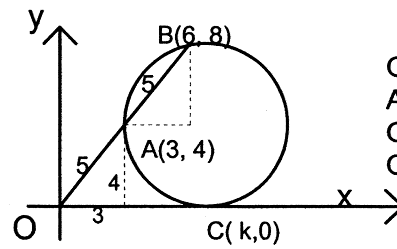
7).



Draw segments parallel to the legs as shown.
A triangle is formed

Since the base angles are complementary, x + y = 90, so the triangle formed is a rt Δ and the line joining the midpoints of the bases is also the median to the hypotenuse of this right triangle. Thus segment *m* is 5 since the length of the median of a rt. Δ = ½ of the hypotenuse so it is ½(10) = 5

8.



OA = 5
AB = 5
OB = 10
OC = k

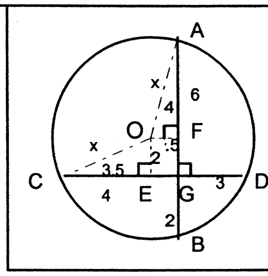
Since (secant)(external segment) = $(\text{tangent})^2$

$(OB)(OA) = (OC)^2$

$k^2 = (10)(5) = 50$,

$k = 5\sqrt{2}$

9.). From center O, draw ⊥ segments to chords $\overline{AB}$ and $\overline{CD}$.



So
CE = ED = 3.5, AF = FB = 4.
Thus EG = 3.5 – 3 = 0.5
Since EG = OF, OF = 0.5
Also FG = OE = 4 – 2 = 2

$(AO)^2 = \left(\frac{1}{2}\right)^2 + 4^2$

$(AO)^2 = \left(\frac{1}{4}\right) + 16 = 65/4$ So $AO = \sqrt{65}/2$

Thus the Diameter = $2(AO) = \sqrt{65}$

Alternatively: $(CO)^2 = \left(\frac{7}{2}\right)^2 + 2^2$;

$(CO)^2 = \left(\frac{49}{4}\right) + 4 = 65/4$, and thus $CO = \sqrt{65}/2$,

Diameter = $2(CO) = \sqrt{65}$

10). The area of the Δ = ½ bh

So ½ (15)(56/5) = ½ $(h_2)(14)$ = ½ $(h_3)(13)$

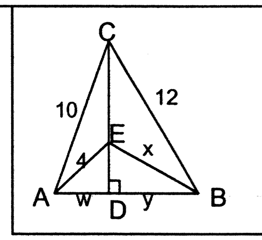
84 = ½ $(h_2)(14)$ and 84 = ½ $(h_3)(13)$

$h_2 = 12$ $h_3 = \dfrac{168}{13}$

Thus $h_2 + h_3 = 12 + \dfrac{168}{13}$

$= \dfrac{156}{13} + \dfrac{168}{13} = \dfrac{324}{13}$

11. Using the Pythagorean Thm:



$10^2 - w^2 = 12^2 - y^2$

and

$4^2 - w^2 = x^2 - y^2$

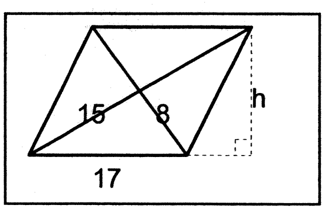
Subtracting these equations, we get

$10^2 - 4^2 = 12^2 - x^2$

So $x^2 = 60$

$x = \sqrt{60} = 2\sqrt{15}$

12.

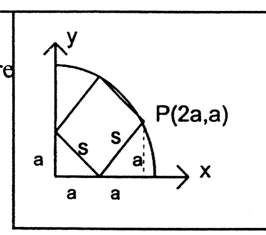


Since the diagonals of a rhombus bisect each other and are ⊥, we have an 8, 15, 17 rt.Δ. Thus the sides of the rhombus are 17. The area of the rhombus = ½ $d_1 \cdot d_2$ = ½ (30)(16) = 240

and The area of the rhombus = $b\,h = 17h$

So $17h = 240$ and $h = 240/17$

13. Area of the square = s^2



Since the inscribed figure is a square of side s, the coordinates of P, a vertex of the square
(Notice, in locating P, two congruent isosceles right triangles are formed.)
Since the radius of circle is 5, and P is on the circle whose equation is $x^2 + y^2 = 5^2$

we get $(2a)^2 + (a)^2 = 5^2$ or $4a^2 + a^2 = 25$

$5a^2 = 25$ so $a^2 = 5$ and so $a = \sqrt{5}$

Since $s = a\sqrt{2}$ the area of the square, $s^2 = \left(a\sqrt{2}\right)^2 = 2a^2$. Thus the square's area is 2(5) = 10

EXTRA ON QUESTION #2

Solving we get:

$AC^2 = 2^2 + 2^2 - 2 \bullet 2 \bullet 2\cos 150^\circ$

$= 4 + 4 - 8\left(-\frac{\sqrt{3}}{2}\right) = 8 + 4\sqrt{3}$

$AC = \sqrt{8 + 4\sqrt{3}}$

EASY!

BUT HOW DO WE GET OUR ANSWER IN THE DESIRED FORM?

Express your answer in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers.

WORKS BACKWARDS (ALWAYS A VIABLE STRATEGY) FROM THE DESIRED FORM UNTIL YOU SEE A CONNECTION!

		Desired	Ours	Connection
AC	=	$\sqrt{a} + \sqrt{b}$	$AC = \sqrt{8 + 4\sqrt{3}}$	?
$(AC)^2$	=	$\left(\sqrt{a} + \sqrt{b}\right)^2$	$(AC)^2 = 8 + 4\sqrt{3}$	?
	=	$\left(\sqrt{a}\right)^2 + 2\sqrt{a}\sqrt{b} + \left(\sqrt{b}\right)^2$		
	=	$a + 2\sqrt{ab} + b$	$= 8 + 2\sqrt{12}$	YES! $a+b = 8$, $ab = 12$

So our desired a and b values must be 6 and 2

And $AC = \sqrt{6} + \sqrt{2}$

Note: $\left(\sqrt{6} + \sqrt{2}\right)^2 = \sqrt{6}^2 + 2\sqrt{6}\sqrt{2} + \sqrt{2}^2 = 6 + 2\sqrt{12} + 2 = 8 + 4\sqrt{3} = AC^2$

Try another:

Express $\sqrt{13 + 4\sqrt{10}}$ in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers.